DNTROL: VLADIMIR KOLETIC/VEER; CLASSROO Building a Team of Learners

his article has a twofold purpose: "studio" sessions (see the sidebar on p. 494) so that we might grow in our collective knowledge regarding the teaching and learning of mathematics, and (2) to share what a group of secondgrade students taught us about teaching geometry. The lessons discussed in studio formed the basis of this article and were primarily designed in collaboration with the consultant, coach, and studio teacher, although certain aspects of the lesson, such as opportunities for student discourse, involved the entire community. Ten to fifteen teachers and two or three administrators were involved in studio over five cycles of three consecutive days each during the school year, for a total of fifteen days. The teachers and administrators in the group were different each day but predominantly spanned kindergarten through grade 5 and came from different schools in the same district. The particular group of students

Teachers use "studio" time to review videotaped classroom episodes, learn about students' developmental levels in geometry, and increase students' use of mathematical language and vocabulary.

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(1) to share the power of opening the classroom door to a community of professional learners through

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discussed here had been working on partner talk and active listening, and they were constantly pressed by the studio teacher (Susan) and the coach (Connie) to share their thinking with one another. The following episode happened during our second studio cycle. All names used in the article are pseudonyms.

The young mathematicians in this studio class were about to begin a unit in geometry using the Bridges in Mathematics curriculum (Snider and Burk 1999). They had just completed an assessment of their knowledge of triangles and rectangles using an assessment sheet from the curriculum. Susan wanted to get a sense of what her students already knew before starting the unit, so she asked, "What do you know about shapes?" and posted their responses. This discussion took place the day before our second studio cycle. Their responses appear below.

What we know about shapes

- "Some shapes have corners; some shapes have none. Smooth equals no points."
- "Some shapes are long—squares are short; rectangles are long."
- "Some shapes are small, like some squares."
- "Shapes have corners."
- "All shapes have straight lines except for circles and ovals."
- "Rectangles and squares and triangles and hexagons and trapezoids and rhombuses are shapes, except circles are not and ovals are not."

A buzz of student voices erupted in the classroom at the last response, which had obviously put many students in a state of mathematical disequilibrium. The children were not quite sure about what their peers were saying. Was it true that circles and ovals are not shapes or that all shapes have corners? At this very moment, the bell rang for lunch, and the teacher had to stop the discussion.

I was the studio consultant at the time and was able to observe this exchange between Susan and her students as well as record the discussion for the teachers and administrators to watch later. The students' responses fascinated me, and I encouraged Susan to continue the conversation after lunch. We were curious whether more dialogue would produce any resolution. Susan and I also talked about the



Capturing authentic classroom instruction on video supplies a common experience to explore in a professional learning community.

value of an open-ended discussion, such as this one, where mathematical misconceptions are often uncovered.

It can be unnerving to teachers when students share misconceptions in mathematics; they often feel a need to correct students on the spot instead of letting them work through their own conceptual misunderstanding. Chapin and her colleagues (2009) remind us that in the process of making sense of experience, "students often generalize ideas in incomplete ways" (p. 61). Moving beyond viewing errors through the lens of diagnosis and remediation, Borasi (1996) advocates that we use errors for purposes of mathematics exploration, problem solving, and reflection-for teachers as well as students. Creating those teachable moments, Boaler and Humphreys (2005) made purposeful use of student errors so everyone would learn more mathematics. Ultimately, mathematics teachers want students to view math as a sense-making experience (Hiebert et al. 1997).

When the children returned from lunch, Susan told them she wanted to hear more of their thinking about what Finn and Georgia had said. Was it true that circles and ovals are not shapes? The discussion continued:

- "If you trace a circle, you can keep on tracing it to infinity."
- "Ovals and circles do not have pointy edges or straight lines."
- "Circles and ovals are not shapes because they do not have edges."
- "I think they should be in with the other shapes because they do not have sides but are still in the group."
- "They do have lines, but they are curved."

After soliciting these responses, Susan asked students to talk to their partner about their thinking of what makes a shape. When Susan pulled students back into a large group, Jenny said she had something important to say. She turned to face her peers and adamantly stated, "If we didn't have shapes, we wouldn't have circle eyes. We would have square heads, triangle tongues, and rectangle bodies. And that doesn't make much sense! We'd look silly."

It was a fascinating, lively discussion, definitely prompting uneasiness in the room, but Susan had no more time to spend on it. Sitting in the back of the room, I really wanted to have a sense of how many students actually believed circles and ovals are not shapes. Toward the end of the discussion, I ran up to Susan and whispered in her ear to ask the students this question directly. She requested that they lower their heads because she was going to ask a thumbs-up or thumbs-down question. After she had asked the question, the thumbs indicated that all the students except Finn and Georgia believed that circles and ovals are shapes.

Hearing this discussion triggered in my own mind the value of teachers having an understanding of the van Hiele theories on developmental levels of geometric thought. I decided to use this video and the students' assessments as a way to gain entry into a deeper understanding of this important model. As a mathematics methods instructor for preservice teachers at a university, I had always talked to my prospective elementary school teachers about the van Hiele developmental levels in geometry. The preservice teachers' mathematics textbook (Van de Walle, Karp, and Bay-Williams 2010) also included a discussion of these levels. I had read about the levels, taught the levels, and even engaged some younger students in some of the activities that go with the various developmental levels, but not until I spent time with these second graders did I have a real sense of these levels coming alive in the classroom.

Five levels in the van Hiele model are considered both sequential and hierarchical (see the **online appendix**). Although the original van Hiele levels were numbered the levels 0-4, Americans started numbering the levels 1-5 instead (Clements and Battista 1992), allowing for prerecognition to be called level 0. Students at this level notice only a subset of the visual characteristics of a shape, resulting in an inability to distinguish between figures (Mason 1998).

During our studio time in the next three days, I shared the videotaped discussion with the professional learning community, and we

"I thought, I learned, I walked away a better teacher. What could be better than that?"

talked about what was happening in the students' statements. Although we knew we were making inferences about their mathematical understanding, the main point the participants brought up was the students' confusion regarding what a polygon is and what a shape is (in general). For the elementary school teachers, this discussion was powerful, and it appeared to strengthen their own mathematical understanding of these concepts. In fact, one of the teachers came right out and asked what the definition of a polygon is. We also talked about the students' statements and which ones had the potential of leading to mathematical misconceptions and errors in student reasoning in mathematics, such as the idea of a curved line. This discussion exposed some of the teachers' own misconceptions related to the topic.

After the discussion, we shared the students' assessments of the geometric activity sheets on triangles and rectangles, which had come with the curriculum. In pairs, teachers and administrators reviewed the students' work. They were to identify potential understanding, misconceptions, and errors and see whether they could ascertain into which van Hiele level the students most likely fell, given their work and what evidence they had for their statements. Additionally, they were to identify at least one assessment question they would want to ask the student if they were to have the opportunity to do so. They were amazed that the students appeared to range anywhere from level 0 to level 2, and that even Andy, who had been home schooled and had the reputation of being among the most mathematically gifted in the classroom, did not know what a triangle is.

One teacher wrote the following regarding this experience:

I teach the same grade level and use the same curriculum. I have read about the five [van Hiele] levels in the book and have

Polishing your practice

Studio is a time for teachers and administrators to come together with a consultant and teacher coach as a professional learning community—to do mathematics together; to explore a math lesson in depth, paying particular attention to the "big ideas" of the lesson; and then to actually enact a mathematics lesson in the studio teacher's classroom. The studio model applies elements of Japanese lesson study in that it involves collaborative planning as well as observation, enactment, and analysis of a lesson. Unlike lesson study, studio work focuses the studio community's attention on public work with students as a way to rehearse and refine teaching routines in mathematics. The emphasis in studio is on polishing practice rather than polishing a whole lesson (TDG 2010). Lesson revision, often an element of lesson study (Fernandez and Yoshida 2004), is not a part of the studio experience.

One major premise of studio work is the notion that orchestrating productive mathematical discourse "increases students' opportunities to learn and, in turn, raises achievement and participation levels in mathematics" (TDG 2010, p. 4). During studio lessons, we worked on high-leverage, mathematically productive teaching routines (Ball et al. 2009; Lampert et al. 2010; Webb et al. 2008). Teachers and administrators could observe the lesson, paying particular attention to students' mathematical thinking and the impact of certain teacher moves on creating opportunities for student discourse. One of the goals for teachers and administrators when they observed the studio lesson was to capture as much student discourse as possible. Artifacts from this work became the evidence base for lesson reflection around student thinking.

> skimmed through them. I don't remember one bit about them. Then, today it hit me. I became aware of the developmental levels, and I had the opportunity to talk about them with other people, then look at the students' work and hear their discussions. I think that our students have great number sense, but not geometry sense. Now I realize one size does not fit all.

Another teacher commented,

Reading about the research on the levels was an eye opener—I can see it in my fourth graders. It makes sense now. Sometimes I assume kids know things. I realize I need more informal types of assessments to inform my practice rather than just assuming.

I wrote the following in my journal:

I'm amazed at how the van Hiele levels have just jumped out at us through our work analyzing students' discourse and observing students and looking at their work. I'm amazed at how quickly students pick up on things. I do believe teachers and administrators will have a different way of looking at students' developmental levels in geometry now—not thinking in terms of right or wrong, but how students' understandings are just different developmentally, based on their experiences.

During all three days, this discussion and the studio lessons also generated an important side conversation around the use of mathematics vocabulary and language with second graders. The topic arose when the teachers and administrators looked over the assessments and several of them noticed that not a single student in this class circled squares on the sheet related to rectangles. Moreover, many students defined rectangles as having four sides with two sides longer than the other two. This realization led to a mathematically productive discussion about whether squares are actually rectangles, which uncovered a misconception on the part of several teachers. The teachers then talked about the importance of their mathematical content knowledge and the impact of their own misconceptions on their students' depth and accuracy of mathematical understanding. What followed was another rich discussion about using vocabulary and definitions with second graders. Many of the teachers believed that second graders are not developmentally ready to think of squares as special types of rectangles, yet others claimed their kindergartners could understand this nesting of shapes and had no problem with it. The latter stated that we do not have high enough expectations of our students.

Another situation related to language happened after the studio lesson the first day. I made the following entry in my journal:

Connie talked to me about Susan's use of the term *fancy words* with her students for mathematical terms and how it bothered her. Connie said, "Why not just refer to them as *mathematical terms* or *the terms mathematicians use*?" So, do I address this issue with Susan?

It so happened that I did not need to raise the issue at all. During our debriefing after the studio lesson, another teacher brought up this dilemma, leading to an important discussion about mathematical language and vocabulary. Teachers stated that primary-level students



were quite capable of handling the correct mathematical vocabulary and really seemed to enjoy learning the mathematical terms. In fact, a first-grade teacher said her students wanted the words on their spelling list. "Why *wouldn't* you teach them the correct terms?"

At the end of each day, studio participants answered the following reflection question: "What are key elements of your professional learning from today's collaborative inquiry?" Many of their responses reflect evidence of the impact this discussion had:

- "We need to be more mindful in our own teaching and how we use terms. We need to make sure that even first graders are aware of different types of triangles."
- "Our use of vocabulary and/or definitions within the classroom is also very important. It can lead to student understanding and/ or misconceptions. We should emphasize the use of correct mathematical vocabulary and definitions. This will assist in students' mathematical understandings and give us an accurate and solid foundation to stage our lessons."
- "That vocabulary is a big and integral part of math. Teaching and thinking about math terms are important at this young age. I need to be using more math vocabulary on a regular basis. When we get to our geometry unit, I will really focus on vocabulary. I want my first graders going into second grade with some knowledge of polygons, etc."
- "Vocabulary and definitions. It is important that students have a shared understanding and develop accurate definitions for these terms. It is not ideal that we give them the definitions but [we should] allow them to explore and develop their own. This helps in their development of justification and generalization skills."

At the end of the cycle, Susan shared that "in the moment, sometimes it is really difficult to process what students are saying and to use that information to guide the lesson." As she reflected on her studio teaching, she said, "I need to continually remind myself that things do not have to be perfect. The purpose is to be in the moment and listen to what the kids are saying." Susan gave us a gift by opening up her classroom door and letting us be observers and learners in her classroom. We cannot stress enough the value of teachers decentralizing their teaching so that others can learn.

Professional development in mathematics through studio work is an amazing opportunity to grow as a learning community. It involves grappling with research in mathematics, doing mathematics, trying out high-leverage practices with students, and then reflecting on it all with colleagues. When you couple these undertakings with access to "in the moment teaching" and student thinking through mathematical discourse, potential exists for powerful learning to happen. As one teacher wrote at the end of her reflection, "I thought, I learned, I walked away a better teacher. What could be better than that?"

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The "studio" professional development model is an opportunity for teachers to polish their instructional practice.



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